

Fair Division in the Middle School

by James Kinyon

After taking a summer discrete mathematics institute aimed at high school teachers, I set out to use fair division in my "Eighth-Grade Mathematics" course. I teach four sections to about 100 students total per year. The setting is a rural/suburban school, Roland-Story Middle School, near Ames, Iowa. I used the module, *Fair Divisions: Getting Your Fair Share* [1], as a resource.

I began by telling the students that, later in the term, several cakes would be delivered, which we'd have to divide fairly among the class. I defined a "fair division method" for a group to be any method that everyone in the group agrees (in advance) is fair. Some of the students knew of the "divide-and-choose" method for two, which we all agreed was fair. Then, for three days, groups of three or four students came up with and tried methods for fairly dividing pictures of round cakes, square cakes, and irregularly-shaped cakes. (See [1], and [2], p. 76-82.)

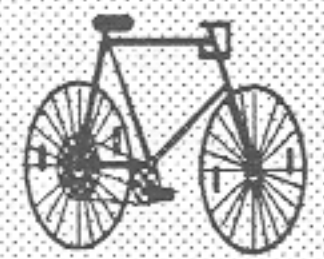
One Friday, our parent support group provided a rectangular and a round cake for each of my classes. The students suddenly became vitally concerned with the fairness of the division! In fact, one student decided that since HE was the biggest kid and HE had the knife, HE should take the biggest piece! (A brief intervention by the teacher was necessary to review the meaning of fairness.)

The students in my room sit at six tables in groups of three or four. We decided to first divide each of the cakes fairly into six pieces, one for each table. I suggested that we first elect two "designated cutters" to split each cake in half, then elect four cutters from the remaining tables to cut each half in thirds. When this was done, each sixth of a cake was cut fairly by the students at each table (using any method they had agreed on).

After the fair division of divisible (continuous) objects, we went on to dividing indivisible (discrete) objects. I first created a scenario in which a student from a math club left for Tibet, leaving some of his possessions to be divided among the rest (see sidebar). Students first discussed the problem in groups of two or three, and tried to devise their own methods. I then introduced the method of "sealed bids" (described in [1], p. 29-31 and [2], p. 51-58) and asked students to try the method on the problem. I found it necessary to provide a detailed form to help students organize their data, as in [1], p. 29. This problem worked well as a warm-up, since students tended to know the actual worth of objects like bicycles and stereos. For homework, I gave them an estate division problem with objects that they did not know the value of (a piano, mountain cabin, boat, etc.), and had them investigate typical monetary values as part of the assignment. I also gave students other similar problems (see [1] for ideas).

Students liked the chance of pace for the two weeks spent on the Fair Division unit. Perhaps the most important part turned out to be not the mathematics, but the resulting student discussions on their own understanding of fairness, and their own values. However, during this time, they solved problems, estimated, computed ratios, organized data logically, and did arithmetic with and without calculators; reinforcing many standard topics in the eighth-grade mathematics curriculum.

Fairly Dividing a Set of Gifts



Johnny's dad has taken a job in Tibet and is planning to move the family there. Since Johnny cannot take his larger possessions on the plane, he has decided to give some of them to you, the members of his math club at school. Here are the things that you need to divide fairly among yourselves.

1993 Encyclopedia.
 Nearly-new 12 speed bicycle.
 Collection of 100 Elvis Presley records.
 400-Watt Sony stereo system.
 Macintosh computer from Wal-Mart.

References:

- [1] Bennett, et al., *Fair Divisions: Getting Your Fair Share*, HiMAP Module 9, COMAP, Arlington, MA, 1987.
- [2] Crisler, Fisher, and Froelich, *Discrete Mathematics Through Applications*, COMAP, Freeman, NY, 1994.

Editor's notes: The method of sealed bids was described by the mathematician Hugo Steinhaus, and is often called the "Steinhaus Method". An example appears in "Dear Ann Landers", by Janice Ricks, *In Discrete Mathematics*, #3, Aug. 1993, p. 2.

